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Mark Scheme (Results)
January 2011

GCE

GCE Core Mathematics C3 (6665) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $*$ The answer is printed on the paper
   - $\sqsubset$ The second mark is dependent on gaining the first mark

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January 2011
Core Mathematics C3 6665
Mark Scheme

Question Number	Scheme	Marks
1. **(a)**	$7\cos x - 24\sin x = R\cos(x+\alpha)$ $7\cos x - 24\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ Equate $\cos x$: $7 = R\cos\alpha$ Equate $\sin x$: $24 = R\sin\alpha$ $R = \sqrt{7^2+24^2}\,;= 25$ $\quad R = 25$ $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^c$ $\quad \tan\alpha = \frac{24}{7}$ or $\tan\alpha = \frac{7}{24}$ awrt 1.287 Hence, $7\cos x - 24\sin x = 25\cos(x+1.287)$	B1 M1 A1 (3)
(b)	Minimum value = $\underline{-25}$ $\quad -25$ or $-R$	B1ft (1)
(c)	$7\cos x - 24\sin x = 10$ $25\cos(x+1.287) = 10$ $\cos(x+1.287) = \frac{10}{25}$ $\quad \cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$ PV $= 1.159279481...^c$ or $66.42182152...^\circ$ $\quad$ For applying $\cos^{-1}\left(\frac{10}{\text{their } R}\right)$ So, $x + 1.287 = \{1.159279...^c, 5.123906...^c, 7.442465...^c\}$ $\quad$ either 2π + or − their PV^c or 360° + or − their PV$^\circ$ gives, $x = \{3.836906..., 6.155465...\}$ $\quad$ awrt 3.84 OR 6.16 awrt 3.84 AND 6.16	M1 M1 M1 A1 A1 (5) **[9]**

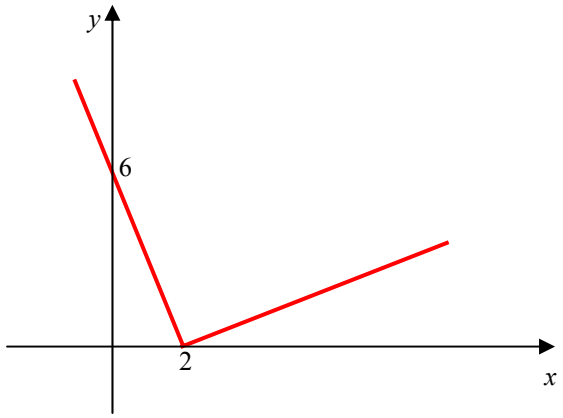
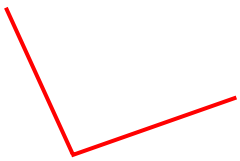
Question Number	Scheme	Marks
2. **(a)**	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$	
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$ An attempt to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$ Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$	A1 (4)
(b)	$f(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, \quad x>1$	
	$f(x) = \dfrac{(4x+1)}{(2x-1)} - 2$	
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$ An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$	
	$= \dfrac{3}{(2x-1)}$ Correct result	A1 * (2)
(c)	$f(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$	
	$f'(x) = 3(-1)(2x-1)^{-2}(2)$ $\pm k(2x-1)^{-2}$	M1
		A1 aef
	$f'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$ Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1 (3) **[9]**

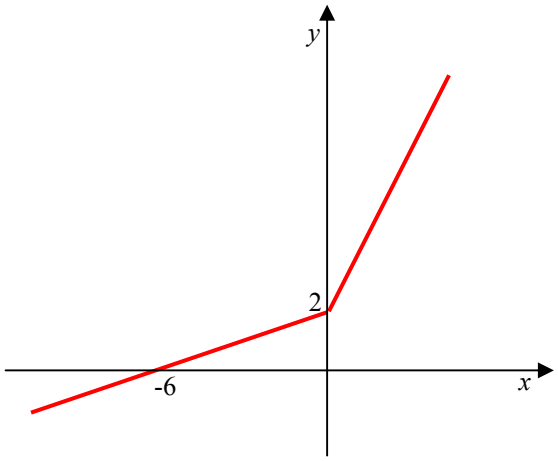
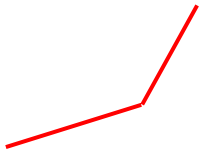
Question Number	Scheme	Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$	
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$ — Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$	
	$4\sin^2\theta - 2\sin\theta - 1 = 0$ — Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ — Applies the quadratic formula. See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$ — Any one correct answer	A1
	$\theta = \{54, 126, 198, 342\}$ — 180-their pv; All four solutions correct.	dM1(*) A1
		[6]

Question Number	Scheme		Marks
4. **(a)**	$\theta = 20 + A\mathrm{e}^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + A\mathrm{e}^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70\mathrm{e}^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70\mathrm{e}^{-k(5)}$	Substitutes $t = 5$ and $\theta = 55$ into eqn *	
	$\frac{35}{70} = \mathrm{e}^{-5k}$	and rearranges eqn * to make $\mathrm{e}^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70\mathrm{e}^{-\frac{1}{5}t\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{1}{5}\ln 2.(70)\mathrm{e}^{-\frac{1}{5}t\ln 2}$	$\pm\alpha\,\mathrm{e}^{-kt}$ where $k = \frac{1}{5}\ln 2$	M1
		$-14\ln 2\,\mathrm{e}^{-\frac{1}{5}t\ln 2}$	A1 oe
	When $t = 10$, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -14\ln 2\,\mathrm{e}^{-2\ln 2}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\min$ (3 dp.)	awrt ± 2.426	A1
			(3)
			[8]

Question Number	Scheme		Marks
5. **(a)**	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x)=0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of {x}=1 OR x={8}	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1,\{0\})$ and $B(8,\{0\})$	B1
			(2)
(b)	Apply product rule: $\left\{\begin{array}{ll} u = (8-x) & v = \ln x \\ \frac{du}{dx} = -1 & \frac{dv}{dx} = \frac{1}{x} \end{array}\right\}$	$vu' + uv'$	M1
	$f'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $f'(3.5)$ and $f'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $f'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $f'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55)+1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both $x_1 =$ awrt 3.529 and $x_2 =$ awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3) **[13]**

Question Number	Scheme		Marks
6. **(a)**	$y = \frac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \frac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \frac{3+5x}{x+2}$	$\frac{3+5x}{x+2}$	A1 oe (3)
(b)	Range of g is $-9 \leq g(x) \leq 4$ or $-9 \leq y \leq 4$	Correct Range	B1 (1)
(c)		Deduces that g(2) is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	$fg(8) = f(4)$	Correct order g followed by f	M1
	$= \frac{3-4(2)}{4-5} = \frac{-5}{-1} = 5$	5	A1 (2)
(e)(i)		Correct shape  $(2,\{0\}),(\{0\},6)$	B1 B1

Question Number	Scheme	Marks
(e)(ii)	 Correct shape  Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 B1 (4)
(f)	Domain of g^{-1} is $-9 \leq x \leq 4$	Either correct answer or a follow through from part (b) answer B1$\sqrt{}$ (1) **[13]**

Question Number	Scheme		Marks
7 **(a)**	$y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$	Applying $\frac{vu' - uv'}{v^2}$ Any one term correct on the numerator Fully correct (unsimplified).	M1 A1 A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$.	
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)	No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin \pi}{2 + \cos \pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $\mathrm{m}(\mathbf{T}) = \dfrac{6\sin \pi + 4\cos \pi + 2}{(2 + \cos \pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\mathrm{m}(\mathbf{T}) = -2$	B1
	Either $\mathbf{T}$: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	$\mathbf{T}$: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

Question Number	Scheme	Marks
8. **(a)**	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$	
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$ $-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	M1 A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$ Convincing proof. Must see both underlined steps.	A1 **AG** **(3)**
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$	
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$ $K\sec 2y \tan 2y$ $2\sec 2y \tan 2y$	M1 A1 **(2)**
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$ Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$ Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$ Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$	
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$ $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1 **(4)** **[9]**

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